Padasalai’s Telegram Groups!

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PART – A (20x1=20)

1. If \( A \) is a 3x3 non – singular matrix such that \( AA^T = A^T A \) and \( B = A^{-1} A^T \) then \( BB^T \) is
   (1) \( A \) (2) \( B \) (3) \( I \) (4) \( B^T \)

2. If \( A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \) and \( A(adj A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \) then \( k \) is
   (1) 0 (2) \( \sin \theta \) (3) \( \cos \theta \) (4) 1

3. The conjugate of a complex number is \( \frac{1}{i-2} \) then the complex number is
   (1) \( i+2 \) (2) \( \frac{-1}{i+2} \) (3) \( \frac{-1}{i-2} \) (4) \( \frac{1}{i-2} \)

4. If \( \alpha \) and \( \beta \) are the roots of \( x^2 + x + 1 = 0 \) then \( \alpha^{2020} + \beta^{2020} \) is
   (1) \( -2 \) (2) \( -1 \) (3) 1 (4) 2

5. A polynomial equation in \( x \) of degree \( n \) always has
   (1) \( n \) different roots (2) \( n \) real roots (3) \( n \) imaginary roots (4) at most one root

6. The polynomial \( x^3 - kx^2 + 9x \) has three real zeros if and only if \( k \) satisfies
   (1) \( |k| \leq 6 \) (2) \( k = 0 \) (3) \( |k| > 6 \) (4) \( |k| \geq 6 \)

7. \( \sin^{-1}(\cos x) = \frac{\pi}{2} - x \) is valid for
   (1) \( -\pi \leq x \leq 0 \) (2) \( 0 \leq x \leq \pi \) (3) \( -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \) (4) \( -\frac{\pi}{4} \leq x \leq \frac{3\pi}{4} \)

8. If \( |x| \leq 1 \) then \( 2\tan^{-1} x - \sin^{-1} \frac{2x}{1 + x^2} \) is equal to
   (1) \( \tan^{-1} x \) (2) \( \sin^{-1} x \) (3) 0 (4) \( \pi \)

9. If \( x + y - k = 0 \) is a normal to the parabola \( y^2 - 12x = 0 \) then the value of \( k \) is
   (1) 3 (2) \( -1 \) (3) 1 (4) 9

10. If the coordinates at one end of the circle \( x^2 + y^2 - 8x - 4y + c = 0 \) are \((11, 2)\) the coordinates of the other end are
   (1) \((-5, 2)\) (2) \((3, -2)\) (3) \((5, -2)\) (4) \((-3, 2)\)

11. The volume of the parallelepiped with its edges represented by the vectors \( \vec{i} + \vec{j}, \vec{i} + 2\vec{j}, \vec{i} + \vec{j} + \vec{k} \) is
    (1) \( \frac{\pi}{2} \) (2) \( \frac{\pi}{3} \) (3) \( \pi \) (4) \( \frac{\pi}{4} \)

12. The angle between the line \( \vec{r} = (\vec{i} + 2\vec{j} - 3\vec{k}) + t(2\vec{i} + \vec{j} - 2\vec{k}) \) and the plane \( \vec{r}.(\vec{i} + \vec{j}) + 4 = 0 \) is
    (1) 0° (2) 30° (3) 45° (4) 90°

13. The order of the matrix \( A \) is 3 then \( \det(kA) \) is
    (1) \( k^3 \det(A) \) (2) \( k^2 \det(A) \) (3) \( k \det(A) \) (4) \( \det(A) \)
14. In a system of 3 linear non homogeneous equation with 3 unknowns if \( \Delta = 0 \) and \( \Delta_x \neq 0, \Delta_z = 0 \) then the system has

(1) unique solution
(2) two solutions
(3) infinitely many solutions
(4) no solutions

15. If \( P \) represents the variable complex number \( z \) and if \( \left| 2z - 1 \right| = 2 \left| z \right| \) then the locus of \( P \) is

(1) the straight line \( x = \frac{1}{4} \)
(2) the straight line \( y = \frac{1}{4} \)
(3) the straight line \( z = \frac{1}{2} \)
(4) the circle \( x^2 + y^2 - 4x - 1 = 0 \)

16. A polynomial \( P(x) \) of degree \( n \) is said to be a reciprocal polynomial then

(1) \( P(x) = x^n P\left(\frac{1}{x}\right) \)
(2) \( P(-x) = x^n P\left(\frac{1}{x}\right) \)
(3) \( P(x) = x^n P\left(-\frac{1}{x}\right) \)
(4) \( P(-x) = x^n P\left(-\frac{1}{x}\right) \)

17. The discontinuous points of \( y = \tan x \) is

(1) \( x = (2n - 1)\frac{\pi}{2}, n \in Z \)
(2) \( x = (2n + 1)\frac{\pi}{2}, n \in Z \)
(3) \( x = (2n - 1)\frac{\pi}{4}, n \in Z \)
(4) \( x = (2n + 1)\frac{\pi}{4}, n \in Z \)

18. The eccentricity of the hyperbola whose latus rectum is equal to half of its conjugate axis is

(1) \( \frac{\sqrt{3}}{2} \)
(2) \( \frac{5}{3} \)
(3) \( \frac{3}{2} \)
(4) \( \frac{\sqrt{5}}{2} \)

19. If \( \vec{a} + \vec{b} + \vec{c} = 0, \left| \vec{a} \right| = 3, \left| \vec{b} \right| = 4, \left| \vec{c} \right| = 5 \) then the angle between \( \vec{a} \) and \( \vec{b} \) is

(1) \( \frac{\pi}{6} \)
(2) \( \frac{2\pi}{3} \)
(3) \( \frac{5\pi}{3} \)
(4) \( \frac{\pi}{2} \)

20. \( \vec{r} = 3\vec{i} + \vec{j} \) is the equation of

(1) a straight line joining the points \( \vec{r} \) and \( \vec{j} \)
(2) \( xy \) plane
(3) \( yoz \) plane
(4) \( zox \) plane

PART – B (7x2=14)
Answer any 7 of the following questions. Q.No 30 is compulsory

21. Prove that \( \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \) is orthogonal

22. Prove \( z \) is real if and only if \( z = \overline{z} \)

23. Show that the equation \( 2x^2 - 6x + 7 = 0 \) cannot be satisfied by any real values of \( x \)

24. State the reason for \( \cos^{-1} \left[ \cos \left( -\frac{\pi}{6} \right) \right] \neq -\frac{\pi}{6} \)

25. If \( y = 4x + c \) is a tangent to the circle \( x^2 + y^2 = 9 \) find \( c \)

26. A particle acted on by constant forces \( 8\vec{i} + 2\vec{j} - 6\vec{k} \) and \( 6\vec{i} + 2\vec{j} - 2\vec{k} \) is displaced from the point \((1, 2, 3)\) to the point \((5, 4, 1)\) find the total work done by the forces.
27. Solve by determinant method \( x - y = 2, \ 3y = 3x - 7 \)
28. Identify the type of conic section \( x^2 + y^2 + x - y = 0 \)
29. Find the distance of a point \((2, 5, -3)\) from the plane \( \vec{r} \cdot (6\vec{i} - 3\vec{j} + 2\vec{k}) = 5 \)
30. What is the principal value ? and find the principal value of \( \tan^{-1} \left( \frac{-1}{\sqrt{3}} \right) \)

### PART – C (7x3=21)
Answer any 7 of the following questions. Q.No 40 is compulsory

31. Reduce the matrix \[
\begin{bmatrix}
0 & 3 & 1 & 6 \\
-1 & 0 & 2 & 5 \\
4 & 2 & 0 & 0 \\
\end{bmatrix}
\]
to a row – echelon form.

32. Show that the equation \( z^3 + 2z = 0 \) has five solutions.

33. If \( p \) is real discuss the nature of the roots of the equation \( 4x^2 + 4px + p + 2 = 0 \) in terms of \( p \)

34. Solve \( \tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4} \), if \( 6x^2 < 1 \)

35. A search light has a parabolic reflector ( has a cross section that forms a bowl). The parabolic bowl is 40 cm wide from rim to rim and 30 cm deep. The bulb is located at the focus. (1) What is the equation of the parabola used for reflector?
(2) How far from the vertex is the bulb to be placed so that the maximum distance covered?

36. With usual notations in any triangle \( ABC \), prove by vector method that
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

37. Find the value of \[
\left( \frac{1 + \sin \frac{\pi}{10} + i \cos \frac{\pi}{10}}{1 + \sin \frac{\pi}{10} - i \cos \frac{\pi}{10}} \right)^{10}
\]

38. Find the tangent equation of the parabola \( y^2 = 12x \ at \ t = 1 \)

39. Determine the number of positive and negative roots of the equation
\( x^9 - 5x^8 - 14x^7 = 0 \)

40. Show that the area of the parallelogram having diagonals \( 3\vec{i} + \vec{j} - 2\vec{k} \) and \( 6\vec{i} - 3\vec{j} + 4\vec{k} \) is \( 5\sqrt{3} \)

### PART – D (7x5=35)
Answer all the questions

41. a) Use matrix inversion method to solve the problem. The prices of three commodities A, B and C are \( \text{₹} \ x, y \) and \( z \) per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process P, Q and R earn \( \text{₹} \ 15,000, \text{₹} \ 1,000 \) and \( \text{₹} \ 4,000 \) respectively. Find the prices per unit of A, B and C.

b) Investigate the values of \( \lambda \) and \( \mu \) the system of linear equations
\[
2x + 3y + 5z = 9, \ 7x + 3y - 5z = 8, \ 2x + 3y + \lambda z = \mu
\]
(i) no solution (ii) a unique solution (iii) an infinite number of solution
42. a) Show that (i) \( (2 + i \sqrt{3})^{10} - (2 - i \sqrt{3})^{10} \) is purely imaginary
    (ii) \( \left( \frac{19 - 7i}{9 + i} \right)^{12} + \left( \frac{20 - 5i}{7 - 6i} \right)^{12} \) is real.

    OR

b) If \( z = x + iy \) and \( \arg \left( \frac{z - 1}{z + 1} \right) = \frac{\pi}{2} \) show that \( x^2 + y^2 - 1 = 0 \)

43. a) If the equations \( x^2 + px + q = 0 \) and \( x^2 + p'x + q' = 0 \) have a common root, show
    that it must be equal to \( \frac{pq' - p'q}{q - q'} \) or \( \frac{q - q'}{p' - p} \)

    OR

b) Solve \( 8x^{3/2} - 8x^{1/2} = 63 \)

44. a) (i) Find the domain of \( f(x) = \sin^{-1} \left( \frac{|x| - 2}{3} \right) + \cos^{-1} \left( \frac{1 - |x|}{4} \right) \)

    (ii) Show that \( \cot^{-1} \left( \frac{1}{\sqrt{x^2 - 1}} \right) = \sec^{-1} x, \ |x| > 1 \)

    OR

b) If \( \cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi \) and \( 0 < x, y, z < 1 \) show that \( x^2 + y^2 + z^2 + 2xyz = 1 \)

45. a) Show that the absolute value of difference of the focal distances of any point \( P \) on the
    hyperbola is the length of its transverse axis.

    OR

b) Two coast guard stations are located 600 km apart at points \( A(0, 0) \) and \( B(0, 600) \). A
    distress signal from a ship at \( P \) is received at slightly different times by two stations. It is
    determined that the ship is 200 km farther from station A than it is from station B.
    Determine the equation of hyperbola that passes through the location of the ship.

46. a) Prove by vector method \( \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \)

    OR

b) Verify \( (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a} \vec{b} \vec{d}] \vec{c} - [\vec{a} \vec{b} \vec{c}] \vec{d} \) for the following vectors
    \( \vec{a} = \vec{i} + \vec{j} + \vec{k}, \ \vec{b} = 2\vec{i} + \vec{k}, \ \vec{c} = 2\vec{i} + \vec{j} + \vec{k}, \ \vec{d} = \vec{i} + \vec{j} + 2\vec{k} \)

47. a) If \( n \) is a positive integer then prove that \( \left( \sqrt{3} + i \right)^n + \left( \sqrt{3} - i \right)^n = 2^{n+1} \cos \frac{n\pi}{6} \)

    OR

b) Find the non–parametric form of vector equation and Cartesian equation of the plane
    passing through the point \( (1, -2, 4) \) and perpendicular to the plane \( x + 2y - 3z = 11 \)
    and parallel to the line \( \frac{x+7}{3} = \frac{y+3}{1} = \frac{z}{1} \)

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